FROM MOTION TO DATA TO KNOWLEDGE

Application Note

Calculating the Velocity of a Ball Moving Toward or Away From a Camera with ProAnalyst®

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Abstract

This application note describes a technique for measuring the velocity of a ball moving in a linear path toward or away from a camera. ProAnalyst is used to automatically track the apparent size of the ball as a function of time. This data is exported to an Excel spreadsheet for computation. The user just needs to provide the real diameter of the ball and the distance from the ball to the camera at the first frame of the video.

Introduction

The purpose of this application note is to find the velocity of a ball based on its apparent change in radius. ProAnalyst is able to calculate the apparent radius of the ball at any given frame (the ball will appear to get larger as it moves closer or smaller as it gets further away). In order to calculate the velocity of the ball from one frame to the next, we need to know the distance the ball travels as well as the time between frames. If **Z** is the distance from the ball to the camera (specifically, we will use the pinhole point of the camera), and **t** is time, then velocity is given by:

$v = \Delta Z/\Delta t$

The time between frames is just the inverse of the frame rate, but the distance the ball travels requires a bit more calculation.

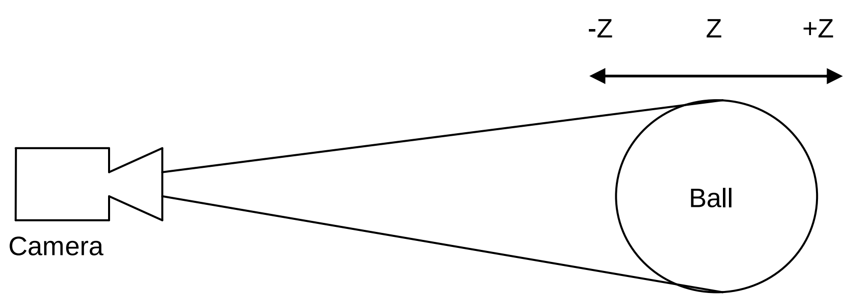

Figure 1. Side View of the Video

A Diagram for the General Case

The first step to solve this problem is to create a diagram for an arbitrary frame of the video (in this case frame x). This diagram uses the pinhole camera model. Here, $\mathbf{P_x}$ is the apparent radius of ball projected onto the image plane of the camera (which can be found for all x), $\mathbf{Z_x}$ is the distance from the ball to the pinhole point of the camera (we only know Z_0), **F** is the distance from the image plane to the pinhole (an unknown constant), and **D** is the diameter of the ball (a known constant). Ultimately, we want to find Z_x as a function of P_x, Z_0, and D.

$Z_x = f\{P_x, Z_0, D\}$

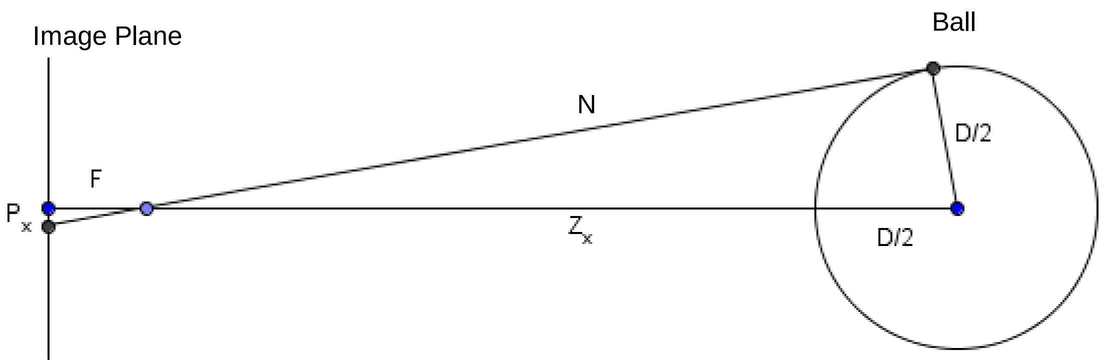

Figure 2. Simple Diagram for Frame x (side view)

Because the image plane is assumed to be perpendicular to the ball's movement, the angle between P_x and F is 90°. Additionally, the line to the surface of the ball is tangent to the ball, and thus perpendicular to its radius. Thus this angle is also 90°. The two angles in the center are vertical angles, and thus are equivalent. We can also find the distance from the pinhole point to its tangent to the ball (N) using the Pythagorean Theorem:

$N = \text{sqrt}((Z_x + D/2)^2 – (D/2)^2)$

$N = \text{sqrt}(Z_x^2 + DZ_x)$

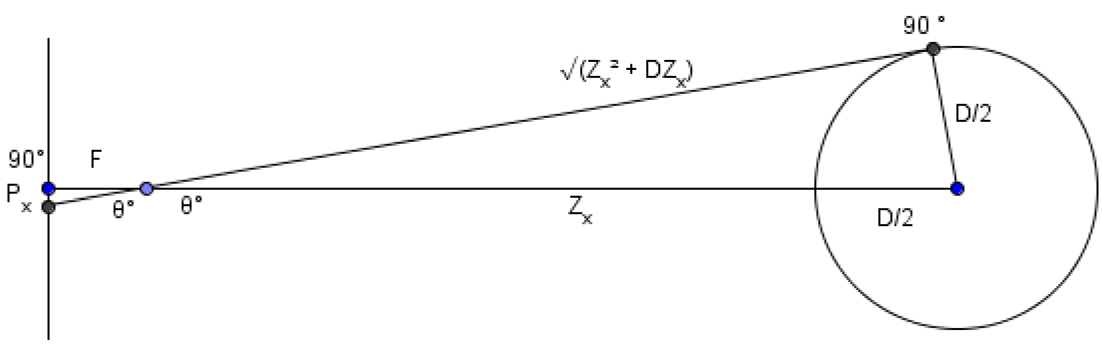

Figure 3. Complete Diagram for Frame x (side view)

Because the two triangles have two equivalent angles, they are similar, and consequently have proportional side lengths. This means:

$\mathrm{sqrt}(Z_x^2 + DZ_x) / F = (D/2) / P_x$

and thus:

$F * (D/2) = \mathrm{sqrt}(Z_x^2 + DZ_x) * P_x$

We want to find Z_x, but we don't know F. We can get around this by looking at frame 0 because we do know Z_0.

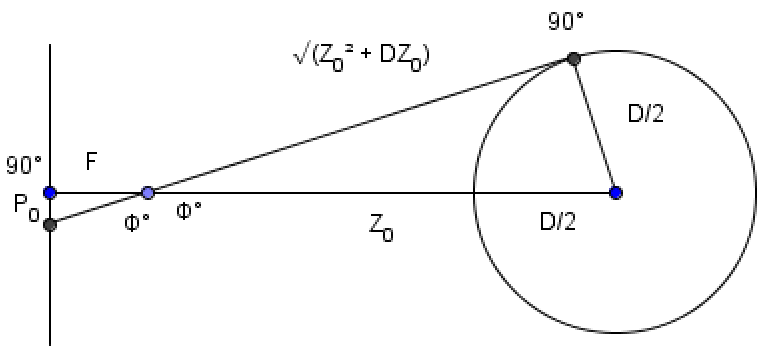

Figure 4. Complete Diagram for Frame 0 (side view)

Using the same calculations as last time, we find:

$F * (D/2) = \mathrm{sqrt}(Z_0^2 + DZ_0) * P_0$

Since both equations equal F * (D/2), they are equal to each other. Thus:

$\mathrm{sqrt}(Z_x^2 + DZ_x) * P_x = \mathrm{sqrt}(Z_0^2 + DZ_0) * P_0$

Now we only have one unknown, so it comes down to solving for Z_x. Since both sides are clearly positive, we can square both sides to get:

$P_x^2 (Z_x^2 + DZ_x) = P_0^2 (Z_0^2 + DZ_0)$

It follows that:

$Z_x^2 + DZ_x - P_0^2 (Z_0^2 + DZ_0)/P_x^2 = 0$

Plugging this in to the quadratic formula:

$$Z_x = (-D + sqrt(D^2 + 4P_0^2(Z_0^2 + DZ_0)/P_x^2))/2$$

Note: $(-D - sqrt(D\text{^}2 + 4P_0^2(Z_0^2 + DZ_0)/P_x^2))/2$ is a negative root of the quadratic equation and is disregarded here.

The change in the ball's distance between frames x and x+1 is given by:

$$\Delta Z_x = (-D + sqrt(D^2 + 4P_0^2(Z_0^2 + DZ_0)/P_{x+1}^2))/2 - (-D + sqrt(D^2 + 4P_0^2(Z_0^2 + DZ_0)/P_x^2))/2$$
$$= (sqrt(D^2 + 4P_0^2(Z_0^2 + DZ_0)/P_{x+1}^2) - sqrt(D^2 + 4P_0^2(Z_0^2 + DZ_0)/P_x^2))/2.$$

The velocity from frame x to frame x+1 is just this number divided by the time between frames:

$$v_x = (sqrt(D^2 + 4P_0^2(Z_0^2 + DZ_0)/P_{x+1}^2) - sqrt(D^2 + 4P_0^2(Z_0^2 + DZ_0)/P_x^2))/(2(t_{x+1} - t_x))$$

Note: A positive velocity for this model means that the ball is moving away from the camera while a negative velocity means it is moving toward the camera.

Creating the Excel Template

An Excel spreadsheet has been created to calculate the ball's velocity between any two frames of the video. The first column is the frame number, the second column is the elapsed time since the start of the video, and the third column is the apparent radius of the ball projected onto the image plane of the camera. There is also a box to enter the diameter of the ball, and a box to enter the starting distance from the ball to the pinhole point of the camera. Using the formula above, the sixth column gives the velocity of the ball from the previous frame to the current frame. There is also a box that displays the average velocity, which is just the average of all of the column 6 values.

The example below shows data from the Excel spreadsheet.

Export of Displayed Graph Data

Column A: Index
Column B: Time
Column C: 1D - Circle - Radius

Enter Measured Ball Diameter: 15.875 mm

Enter Distance from camera to ball in frame 0: 25 mm

Average Ball Velocity: 3.88486 mm/sec

Graph Data

Frame #	Elapsed time (sec)	Apparent radius(mm)	Ball Velocity (mm/sec)
0	0	420.8259583	
1	0.0067	420.5031128	3.555228192
2	0.0133	420.1802368	3.615144286
3	0.02	419.8573914	3.566485424
4	0.0267	419.5345459	3.572133299
5	0.0333	419.2116699	3.632346842
6	0.04	418.8888245	3.583468836
7	0.0467	418.565979	3.589156043
8	0.0533	418.243103	3.649669196
9	0.06	417.9202576	3.600570612
10	0.0667	417.5974121	3.606297517

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